Theory

Q3. Soundness of Resolution:

A sound is provable if the formula is valid, or we can prove the formula using the rules of the system if that formula is true in all interpretations.

To prove that the resolution is sound, we need to show:

Every resolution refutation corresponds to an unsatisfiable set of clauses

We can do this by induction on the length of the resolution refutation.

Base Case:

Resolution refutation length consists of size 1 which is an empty clause, and the empty clause is unsatisfied. Hence the base case is satisfied.

Inductive steps:

Let the refutation be the length of n which corresponds to unsatisfiable sets of clauses and P be the resolution refutation of length *n*+1. Hence P must start with two clauses C1 and C2.

Now by the inductive hypothesis, *C*1​ and *C*2​ correspond to unsatisfiable clauses. Which means there is no interpretation that both *C*1​ and *C*2​ are true.

Now *R* is a resolvent of *C*1​ and *C*2​, that follows that *R* is unsatisfied. This means that the set of clauses {*C*1​,*C*2​,*R*} is unsatisfiable.

Hence, all resolution refutations correspond to unsatisfiable sets of clauses. This means that the resolution is sound.

Completeness of sound:

To prove the system is complete if every valid formula is provable. Or if a formula is true in all cases, then we will be able to prove it using the rules of the system.

To show that the resolution is complete, we must show that every unsatisfiable set of clauses has a resolution refutation.

We can do this by resolving the refutation tree.

The resolution refutation tree is a tree where each node is a clause and the edge represents resolution steps. Now the root of the tree is a set of clauses that we are trying to refute. The leaves of the tree are empty clauses.

We can do this by constructing by starting at the root and repeatedly applying the resolution rule. We will stop when we reach the leaf node which is empty clause.

If the set of clauses are unsatisfied then the resolution refutation tree will reach a leaf node because every unsatisfiable set of clauses has a resolution refutation.

Hence , resolution is complete.